PREDICTION OF STATISTICAL SYSTEM PERFORMANCE FROM PARAMETER DISTRIBUTIONS

by

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United States
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THESIS

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Parameter Distributions

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ABSTRACT

Techniques for extrapolating statistical data on the parameters of individual components to the statistical performance index for an overall system are considered. Two cases are evaluated. (1) The deterministic case in which the system's performance index is known functionally in terms of the system parameters. (2) The non-deterministic case in which only limited data on the performance index and its sensitivity with respect to system parameters is known. Computer programs are developed in both cases for combining given probability density distributions for the parameters into an overall probability density distribution for the system performance index. Theory and programs are developed and verified with a specific numerical example.

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I. INTRODUCTION

The performance of a system, from a statistical point of view, depends upon the nominal values of the parameters of the component parts included in the system and on the statistical distributions of those parameters. Alternately, the response of the component parts to a given signal may be known statistically and it is desired to predict an overall statistical distribution for the system response. Statistical information on the system's response may be interpreted in terms of probability of success or failure (operation or non-operation) for a single system or in terms of the percentage of operational systems that can be expected in an ensemble of systems. Currently three basic methods are being used to obtain performance data for systems; the worst-case method [Mark and Stember 1965], the moment method [Mark and Stember 1965], and the Monte Carlo method [Shreider 1966].

The worst-case method is a non-statistical approach intended to determine whether it is possible, with given parameter tolerance limits, for the systems performance characteristics to fall outside specifications. This method determines nothing of the probability distribution of the performance. Worst-case analysis, although it may be involved, does provide useful engineering insight and is used a great deal in practice. However, its outlook is basically limited.

Statistics are combined with system analysis techniques in the moment method to estimate the probability that performance will remain within specified limits. The actual parameter probability distributions are usually replaced by Gaussian distributions having the same variances. Hence, the moment method has inherent limitations. Parameter probability distribution characteristics such as skewness and peakedness are usually not taken into account.

In the Monte Carlo method a large number of alternate replicas of a system are simulated by mathematical models. Component values are randomly selected and the performance of each replica is determined for its particular set of components. The accuracy of the Monte Carlo method is dependent upon taking a sufficiently large number of replicas and can be both lengthy and costly. Monte Carlo methods are also used extensively in practice although they provide limited engineering information.

The purpose of this thesis is to present a numerical technique for combining statistical data on several independent components into a single distribution function for the system performance. This is accomplished by considering two general cases. The deterministic case, where the functional relationships between the system parameters and the output are known functionally; and the non-deterministic case, where these functional relationships are unknown but measured discrete data is available. In both cases the actual parameter probability distributions are used in conjunction with numerical integration

techniques to obtain the system distribution function. One major advantage of the approach is that the known parameter distributions are quite arbitrary and may be histograms obtained from the results of experiments on individual components.

II. DETERMINISTIC CASE

The object of the deterministic case is to investigate the distribution of a system output when the output has a known mathematical relationship to several internal parameters. The basic assumptions are that all of the internal parameters are independent and that the probability distributions of the internal parameters, although arbitrary, are known mathematically or given in terms of histograms.

To commence the investigation, the general equations for various combinations of parameter distributions which may be expected in a typical system performance index must be obtained. These will basically consist of operations on the distributions of random variables which are interrelated by addition, subtraction, multiplication, and division. The basic mathematics for these operations are well known in the literature, at least in general form [Beckmann 1967].

Consider two independent random variables: X, assuming values x with a known probability density $p_X(x)$, and Y, assuming values y with a known probability density $p_Y(y)$. Also consider the functions $U = U(X, Y)$ and $V = V(X, Y)$ and their inverse functions $X = X(U, V)$ and $Y = Y(U, V)$. Then, when X is near x and Y is near y, U and V must be near $u = u(x, y)$ and $v = v(x, y)$. Hence,

$$p_{XY}(x, y)\, dxdy = p_{UV}(u, v)\, dudv \tag{1}$$

Substituting the inverse functions and the required probability densities, we obtain

$$p_{UV}(u, v) = p_X\,[x(u, v)]\; p_Y\,[y(u, v)] \left| \frac{\partial(x,y)}{\partial(u,v)} \right| \qquad (2)$$

where $p_{UV}(u, v)$ is the joint probability density of U and V, $p_X[x(u, v)]$ is the probability density of X, $p_Y\,[y(u, v)]$ is the probability density of Y, $x(u, v)$ and $y(u, v)$ are the inverse functions, and $\left| \frac{\partial(x,y)}{\partial(u,v)} \right|$ is the Jacobian, given by

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \qquad (3)$$

When X and Y are related by a single functional the foregoing can be modified by letting $U = U(X, Y)$ and $V = X$. (4)

Then,

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & 1 \\ \frac{\partial y}{\partial u} & 0 \end{vmatrix} = - \frac{\partial y}{\partial u} \qquad (5)$$

Equation (2) then becomes

$$p_{UX}(u, x) = p_X(x) p_Y\,[y(u, x)] \left| \frac{\partial y}{\partial u} \right| \qquad (6)$$

Since, in general, $p_U(u) = \int_{-\infty}^{\infty} p_{UX}(u, x)dx$, equation (6) may be written as

$$p_U(u) = \int_{-\infty}^{\infty} p_X(x) p_Y\,[y(u, x)] \left| \frac{\partial y}{\partial u} \right| dx \qquad (7)$$

Using equation (7), the specific formulas of Table I have been compiled for the functional relationships and operations listed, which were chosen on the basis of what might be expected in engineering problems.

The technique outlined here consists of the successive application of the formulas of Table I as required, with the required integration performed numerically. Various types of numerical integration are possible; rectangular integration, trapezoidal integration, or Simpson's Rule. These are summarized as follows:

Rectangular integration [Stanton 1961],

$$\int_{x_1}^{x_n} f(x)\,dx = \Delta x \sum_{i=1}^{n} f(x_i) \tag{8}$$

Trapezoidal integration [Stiefel 1963],

$$\int_{x_1}^{x_n} f(x)\,dx = \Delta x \left\{ \frac{1}{2}\left[f(x_1) + f(x_n) \right] + \sum_{i=2}^{n-1} f(x_i) \right\} \tag{9}$$

Simpson's Rule [Stanton 1961],

$$\int_{x_0}^{x_0+n\Delta x} f(x)\,dx = \frac{\Delta x}{3} \sum_{i=0}^{n} C f(x_i) \tag{10}$$

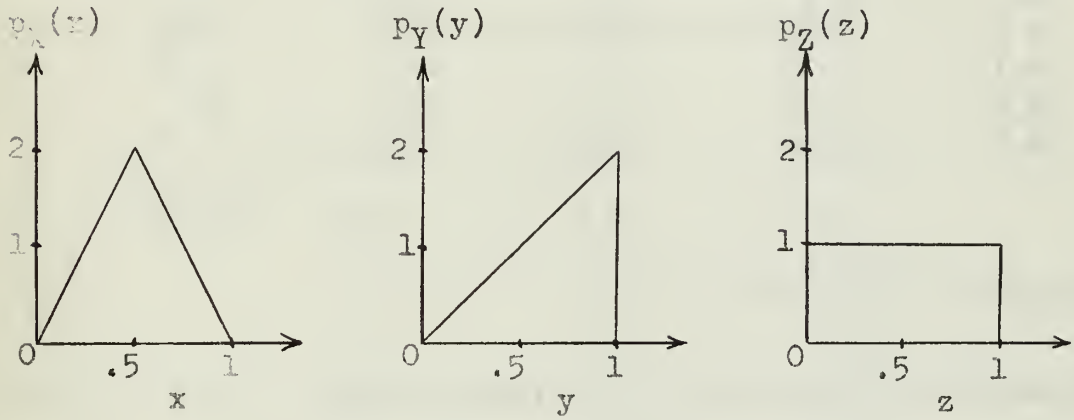
where C = 1, 4, 2, --- 2, 4, 1

Although the specific technique of numerical integration is of prime importance in so far as accuracy and computer programing and time are concerned, the exact procedure used is a detail which is best dictated by the particular problem to which these techniques are applied.

This method may be best illustrated by considering the following example. Consider that:

$$H = \frac{\;}{XY} + b + Z \tag{11}$$

where a and b are constants and X, Y, and Z are independent random variables with known (measured) probability densities. Consider that $p_X(x)$, $p_Y(y)$, and $p_Z(z)$ are given as follows:



(a) Consider first the combination U = XY, with $\Delta x = 0.1$. Using rectangular integration for simplicity, equation (3) of Table I becomes, for $u = u_1$:

$$p_U(u_1) = \int_{-\infty}^{\infty} p_X(x) p_Y(u/x) \frac{dx}{|x|} \cong \Delta x \sum_{i=1}^{9} \frac{p_X(x_i)}{x_i} p_Y\left(\frac{u_i}{x_i}\right) \quad (12)$$

When $u = u_1 = 0.09$

x_i	u_1/x_i	$\frac{p_X(x_i)}{x_i}$	$p_Y(u_1/x_i)$	$\frac{p_X(x_i)}{x_i} p_Y(u_1/x_i)$
0.1	0.9	4.0	1.8	7.2
0.2	0.45	4.0	0.9	3.6
0.3	0.3	4.0	0.6	2.4
0.4	0.22	4.0	0.43	1.72
0.5	0.18	4.0	0.37	1.48
0.6	0.15	2.6	0.30	0.78
0.7	0.13	1.7	0.26	0.44
0.8	0.11	1.0	0.21	0.21
0.9	0.1	0.44	0.20	0.088
			sum =	17.918

$p_U(u_1) = (0.1)(17.918) = 1.79$

when $u = u_2 = 0.2$

x_i	u_2/x_i	$\frac{p_X(x_i)}{x_i}$	$p_Y(u_2/x_i)$	$\frac{p_X(x_i)}{x_i}\, p_Y(u_2/x_i)$
0.1	2.0	4.0	0.0	0.0
0.2	1.0	4.0	2.0	8.0
0.3	0.7	4.0	1.4	5.6
0.4	0.5	4.0	1.0	4.0
0.5	0.4	4.0	0.8	3.2
0.6	0.33	2.6	0.67	1.74
0.7	0.3	1.7	0.6	1.02
0.8	0.25	1.0	0.5	0.5
0.9	0.22	0.44	0.42	0.185
			sum =	24.245

$p_U(u_2) = (0.1)(24.245) = 2.4$

The pattern for evaluating $p_U(u)$ is repeated for $u = .3, .4, --, 1.0$ yielding the curve of Figure 1.

(b) Next consider the function $V = \frac{1}{U} = \frac{1}{XY}$. Here equation (7) of Table I may be applied.

$$p_V(v) = \frac{1}{v^2} p_U(1/v) = u^2 p_U(u) \tag{13}$$

u	v	$p_U(u)$	u^2	$p_V(v)$
0.1	10.0	1.8	0.01	0.018
0.2	5.0	2.42	0.04	0.097
0.3	3.33	2.39	0.09	0.215
0.4	2.5	2.11	0.16	0.338
0.5	2.0	1.54	0.25	0.385
0.6	1.66	1.02	0.36	0.364
0.7	1.43	0.55	0.49	0.270
0.8	1.25	0.28	0.64	0.179
0.9	1.11	0.05	0.81	0.040

$p_V(v)$ is plotted in Figure 2.

(c) The function W may now be calculated using equation (8) of Table I, where

$$W = aV + b = \frac{a}{XY} + b \tag{14}$$

and

$$p_W(w) = \frac{1}{|a|} p_V\left(\frac{w-b}{a}\right) = \frac{1}{|a|} p_V(v) \tag{15}$$

If $a = 2$ and $b = 1$, the following result is obtained.

V	$W = aV + b$	$p_V(v)$	$p_V(v)/2$
1	3	0.0	0.0
2	5	0.385	0.1925
3	7	0.25	0.125
4	9	0.15	0.075
5	11	0.10	0.050
6	13	0.06	0.030
7	15	0.04	0.020
8	17	0.03	0.015
9	19	0.02	0.010
10	21	0.01	0.005

The result is plotted in Figure 3.

(d) Since, from equations (11) and (14),

$$H = W + Z \tag{16}$$

it follows from equation (1) of Table I that $p_H(h)$ is given by the convolution of $p_W(w)$ and $p_Z(z)$. This operation may also be carried out numerically and for the given $p_Z(z)$ may be expressed as

$$\left.\begin{array}{lll} p_H(h) = \int_0^h p_W(w)\,dw & \text{for} & 0 \leq h \leq 1 \\ & & \\ p_H(h) = \int_{h-1}^h p_W(w)\,dw & \text{for} & h > 1 \end{array}\right\} \tag{17}$$

These integrals may also be carried out numerically yielding the results plotted in Figure 4.

To work more complicated problems than the previous example requires a great amount of mathematical calculations. Thus, when using numerical integration techniques the method lends itself nicely to the digital computer. Figures 5 through 9 are the computer flow graphs corresponding to some of the equations of Table I. The computer program for any mathematical function can then be derived from the appropriate combinations of these basic flow graphs.

To illustrate a more complex problem consider the following example·

given

$$\theta = 1285 \left(\frac{1}{A} - \frac{1}{B} \right) + W \tag{18}$$

where

$$A = B_{3a}B_{6a} , \qquad B = B_{3b}B_{6b}$$

and

$$B_{3a} = \frac{B_{3b}}{1 + B_{3b}(2.16 \times 10^{-2})}$$

$$B_{6a} = \frac{B_{6b}}{1 + B_{6b}(0.63 \times 10^{-2})}$$

B_{3b} is a normal distribution; mean = 130, standard deviation = 10

B_{6b} is a normal distribution; mean = 65, standard deviation = 12

W is a uniform distribution from $w = -4/7$ to $w = +4/7$

Theta is system output parameter and the variables B_{3b}, B_{6b}, and W are independent.

Since the variables are independent, equation (18) can first be simplified by ordinary mathematical manipulation.

Let $X = B_{3b}$, $Y = B_{6b}$, $k_1 = 2.16 \times 10^{-2}$, $k_2 = 0.63 \times 10^{-2}$

then $B = XY$

$$A = \frac{XY}{(1 + k_1 X)(1 + k_2 Y)}$$

$$\frac{1}{A} - \frac{1}{B} = k_1 k_2 + \frac{k_1}{Y} + \frac{k_2}{X}$$

and $$\theta = 0.175 + \frac{27.8}{Y} + \frac{8.1}{X} + W \qquad (19)$$

Now, utilizing the basic methods and flow graphs, the problem can be programed on the digital computer and the probability density of θ readily obtained. The program and solution are contained in Appendix I.

III. NON-DETERMINISTIC CASE

In many practical problems the functional relationship between variables is unknown and it cannot be expressed simply by formula. It is generally available only as numerical data from a computer run on a mathematical model of the system under study. Two cases are considered here. First, the case where the system response of interest, U(X), and its sensitivity coefficients, U'(X), are available at discrete values of a single variable, X.

$$U(X) = \left[u(x_1),\ u(x_2),\ ---,\ u(x_n)\right] \tag{20}$$

$$U'(X) = \left[u'(x_1),\ u'(x_2),\ ---,\ u'(x_n)\right] \tag{21}$$

Second, the case where the system response of interest, U(X, Y), and its sensitivity coefficients, U'(X, Y), are available at discrete values of two independent variables, X and Y.

$$U(X, Y) = \begin{vmatrix} u_{11} & u_{12} & --- & u_{1n} \\ u_{21} & u_{22} & --- & u_{2n} \\ - & - & & - \\ - & - & & - \\ u_{m1} & u_{m2} & --- & u_{mn} \end{vmatrix} \tag{22}$$

where

$$u_{ij} = u(x_i, y_j)$$

and

$$U'(X, Y) = \begin{vmatrix} u'_{11} & u'_{12} & \cdots & u'_{1n} \\ u'_{21} & u'_{22} & \cdots & u'_{2n} \\ - & - & & - \\ - & - & & - \\ u'_{m1} & u'_{m2} & \cdots & u'_{mn} \end{vmatrix} \tag{23}$$

where

$$u'_{ij} = \frac{\partial u(x_i , y_j)}{\partial x_i}$$

In both cases the problem reduces to determining $p_U(u)$ from the known distributions $p_X(x)$, $p_Y(y)$ and the data of equations (20) and (21), or (22) and (23).

In the first case, consider that the probability density, $p_X(x)$, and the cumulative distribution,

$$P_X(x_j) = \int_{-\infty}^{x_j} p_X(x)\,dx \tag{24}$$

are known. Then at each data point of u(x),

$$P_U\left[u(x_j)\right] = P_X(x_j) \tag{25}$$

That is, the probability that $u \leq u(x_j)$ is equal the probability that $x \leq x_j$. Equation (25) gives discrete points on the cumulative distribution, $P_U(u)$, for each value of the parameter x for which u(x) has been measured.

It is also well known [Beckman 1967] that

$$p_U(u) = \frac{\Delta P_U(u)}{\Delta u} = \frac{p_X(x)}{\left|\frac{du}{dx}\right|} \tag{26}$$

Thus the slope of the cumulative distribution curve, $\frac{\partial P_U(u_j)}{\partial u_j}$,

at each data point is equal to the known probability density, $p_X(x_j)$, divided by the measured sensitivity function, $\left|\frac{\partial u}{\partial x}\right|$, at the data point. The conclusion is that the cumulative distribution, $P_U(u)$, and its slope may be calculated readily from the measured response and sensitivity data. This curve is therefore evaluated as a piece-wise linear approximation.

When U is a function of two variables, X and Y, the foregoing cumulative distribution curve can be evaluated for each value of the second variable, Y. Thus a family of such curves or data points denoted by $P_U(u, y_j)$ is available. For a specific value of $u = u_i$ each of these curves gives the probability that $u \leq u_i$ when $y = y_j$. The total probability that $u \leq u_i$ depends upon $p_Y(y)\Delta y$ and is given by the equation:

$$P_U(u_i) = \Delta y \sum_{j=1}^{m} P_U(u_i, y_j) p_Y(y_j) \tag{27}$$

where $P_U(u, y_j)$ is the cumulative distribution evaluated with y as a constant and x as a single variable as discussed previously.

This result can be proven by considering the following (see equation (7)):

$$p_U(u) = \int_{-\infty}^{\infty} p_Y(y) p_X\left[x(u,y)\right] \left|\frac{\partial x}{\partial u}\right| dy \tag{28}$$

where $u = u(x, y)$. $\frac{\partial u}{\partial x}$ is the sensitivity function with respect to the parameter x and in equation (28) must be expressed in terms of u and y. It is convenient to denote

$$\frac{\partial u}{\partial x} = u_X(u,y) \tag{29}$$

and then equation (28) becomes

$$p_U(u) = \int_{-\infty}^{\infty} p_Y(y) p_X\left[x(u,y)\right] \frac{dy}{u_X(u,y)} \tag{30}$$

We also know

$$P_U(u) = \int_{-\infty}^{u} p_U(u)du \tag{31}$$

Substituting equation (30) into equation (31) and inter-changing the order of integration yields

$$P_U(u) = \int_{-\infty}^{\infty} p_Y(y)dy \int_{-\infty}^{u} p_X\left[x(u,y)\right] \frac{du}{u_X(u,y)} \tag{32}$$

Now

$$\int_{-\infty}^{u} p_X\left[x(u,y)\right] \frac{du}{u_X(u,y)} = \int_{-\infty}^{u} p_U(u,y)du = P_{UY}(u,y) \tag{33}$$

Since, if Y is considered a constant and X as the random variable, $u = u(x, y)$,

$$p_U(u, y) = \frac{p_X\ [x(x, y)]}{\frac{\partial u}{\partial x}} = \frac{p_X\ [x(x, y)]}{u_X(u, y)} \tag{34}$$

Substituting equation (33) into equation (32) yields:

$$P_U(u) = \int_{-\infty}^{\infty} p_Y(y)P_U(u, y)dy \tag{35}$$

In discrete form, equation (35) becomes equation (27) when rectangular integration is used. However, more accurate integration techniques may also be applied to equation (35).

To demonstrate the application of this non-deterministic method consider part of the previous example of equation (18). It is now assumed that the functional relationship between the output and the parameters is unknown. However, we do know the probability densities of the parameters x and y and have certain measured data which consists of measured values of x, y, and Ø and the corresponding sensitivities for each measured Ø. The data is presented in Table II. Following the procedure outlined, the problem can be programed on the digital computer and the probability density of Ø readily obtained. The program and solution are contained in Appendix II. The results can be compared with those obtained using the deterministic formulas. Noting, of course, that the solution to the deterministic problem,contained in the Appendix, includes the addition of the uniform distribution W(w) and the solution to the non-deterministic problem does not.

IV. CONCLUSIONS

The numerical technique presented here has distinct advantages over the worst-case, the moment, and the Monte Carlo methods. By utilizing the actual parameter probability distributions the results obtained are more accurate than those of the moment method and present a great deal more information than the worst-case method. The accuracy obtained is equivalent to that of a large number of Monte Carlo replicas. Thus the saving in time over the Monte Carlo method is obvious. In general, the numerical method presented here can obtain a more accurate solution in less time than any of the other current methods. However, certain limitations are inherent in the basic assumptions used to formulate the method. Since independent parameters are required, a great number of practical problems can not be handled by this method as it is presented here. By extending the mathematical derivations to include the correlation of parameters, the method could be used for systems in which the parameters are not independent.

Table I

Functional Formulas

(1) $U = X + Y$ $\qquad p_U(u) = \int_{-\infty}^{\infty} p_X(x)p_Y(u-x)dx$

$$= \int_{-\infty}^{\infty} p_X(u-y)p_Y(y)dy$$

(2) $U = X - Y$ $\qquad p_U(u) = \int_{-\infty}^{\infty} p_X(x)p_Y(x-u)dx$

$$= \int_{-\infty}^{\infty} p_X(u+y)p_Y(y)dy$$

(3) $U = XY$ $\qquad p_U(u) = \int_{-\infty}^{\infty} p_X(x)p_Y(u/x)\frac{dx}{|x|}$

(4) $U = X/Y$ $\qquad p_U(u) = \frac{1}{u^2}\int_{-\infty}^{\infty} p_X(x)p_Y(x/u)|x|dx$

$$= \int_{-\infty}^{\infty} p_X(uy)p_Y(y)|y|dy$$

(5) $U = X^2$ $\qquad p_U(u) = \dfrac{p_X(\sqrt{u}) + p_X(-\sqrt{u})}{2\sqrt{u}} \qquad u \geq 0$

(6) $U = |X|$ $\qquad p_U(u) = p_X(x) + p_X(-x) \qquad x \geq 0$

(7) $U = a/X$ $\qquad p_U(u) = \frac{a}{u^2}\, p_X(a/u)$

(8) $U = aX + b$ $\qquad p_U(u) = \frac{1}{a}\, p_X\left(\frac{u-b}{a}\right)$

(a and b are constants)

Table II

Sample Data

x \ y	41	53	65	77	89
110	0.9266	0.7731	0.6761	0.6096	0.5616
120	0.9204	0.7669	0.6699	0.6034	0.5554
130	0.9152	0.7617	0.6647	0.5982	0.5502
140	0.9108	0.7573	0.6603	0.5938	0.5458
150	0.9069	0.7534	0.6564	0.5899	0.5419

θ

y	41	53	65	77	89
$\left\lvert \frac{\partial \theta}{\partial y} \right\rvert$	0.0165	0.00988	0.00658	0.00469	0.00352

x	110	120	130	140	150
$\left\lvert \frac{\partial \theta}{\partial x} \right\rvert$	0.000670	0.000561	0.000478	0.000413	0.000359

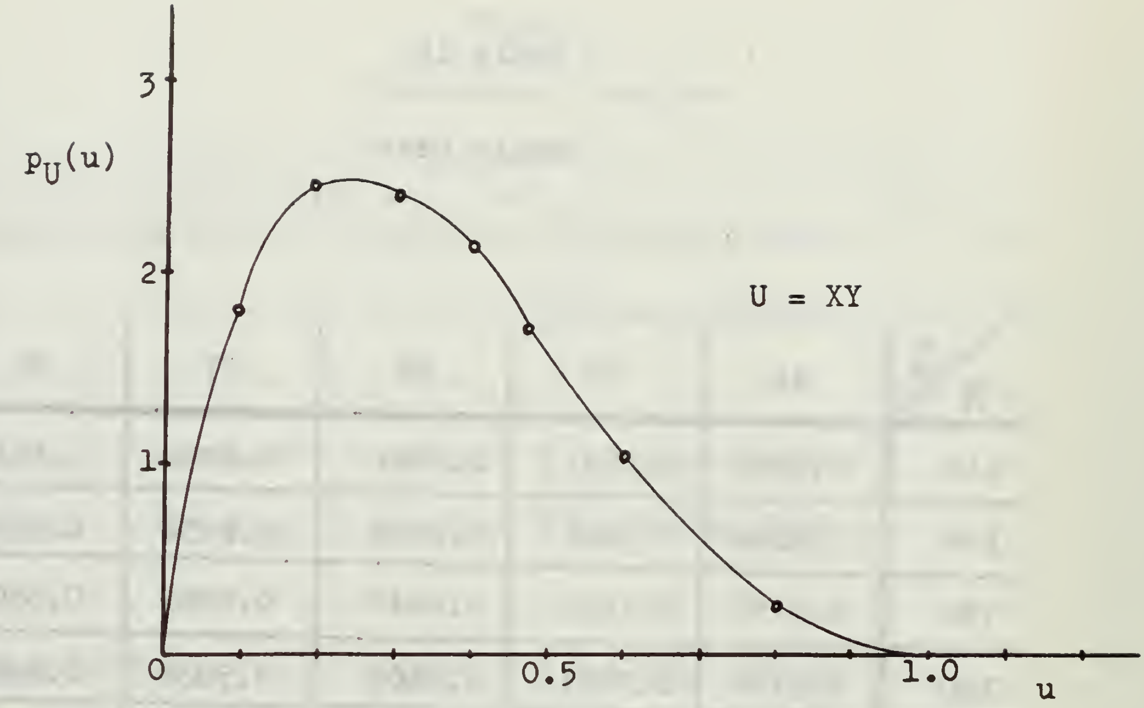


Figure 1

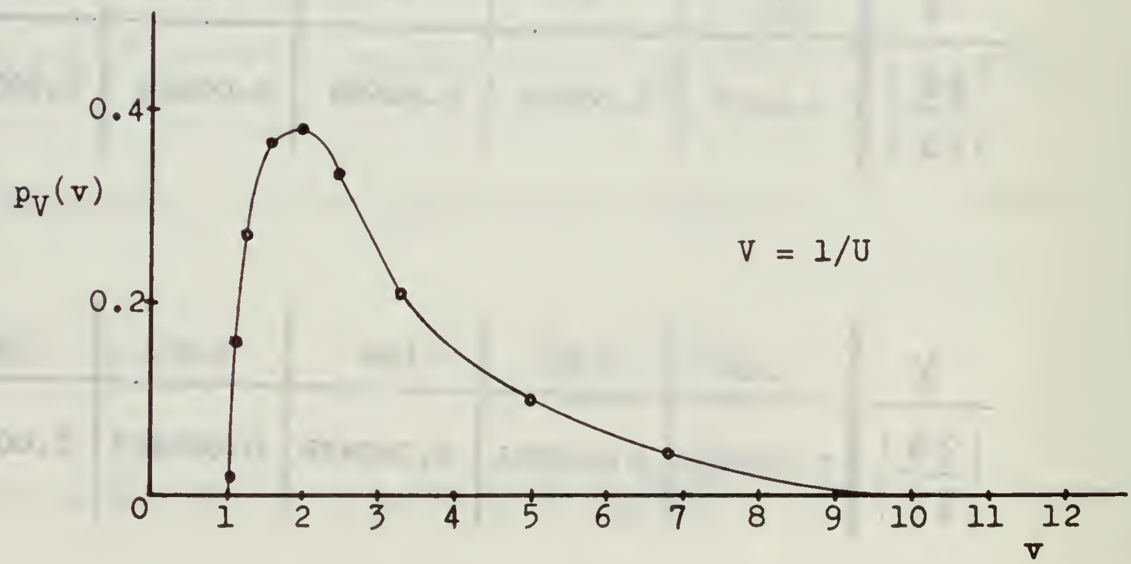


Figure 2

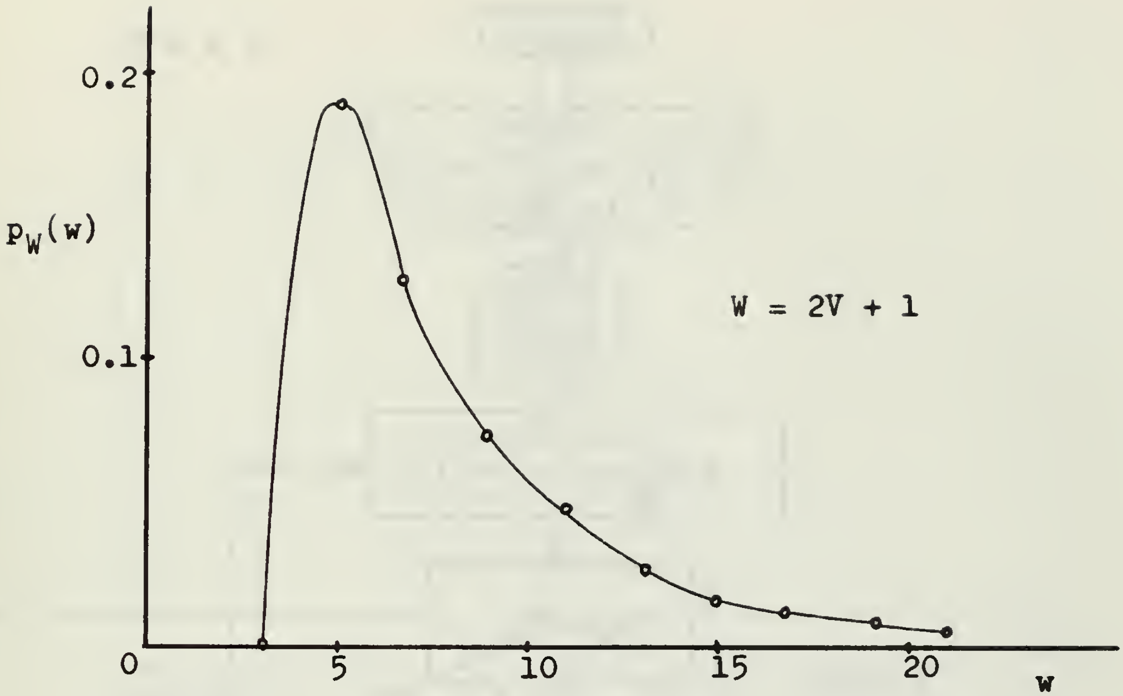


Figure 3

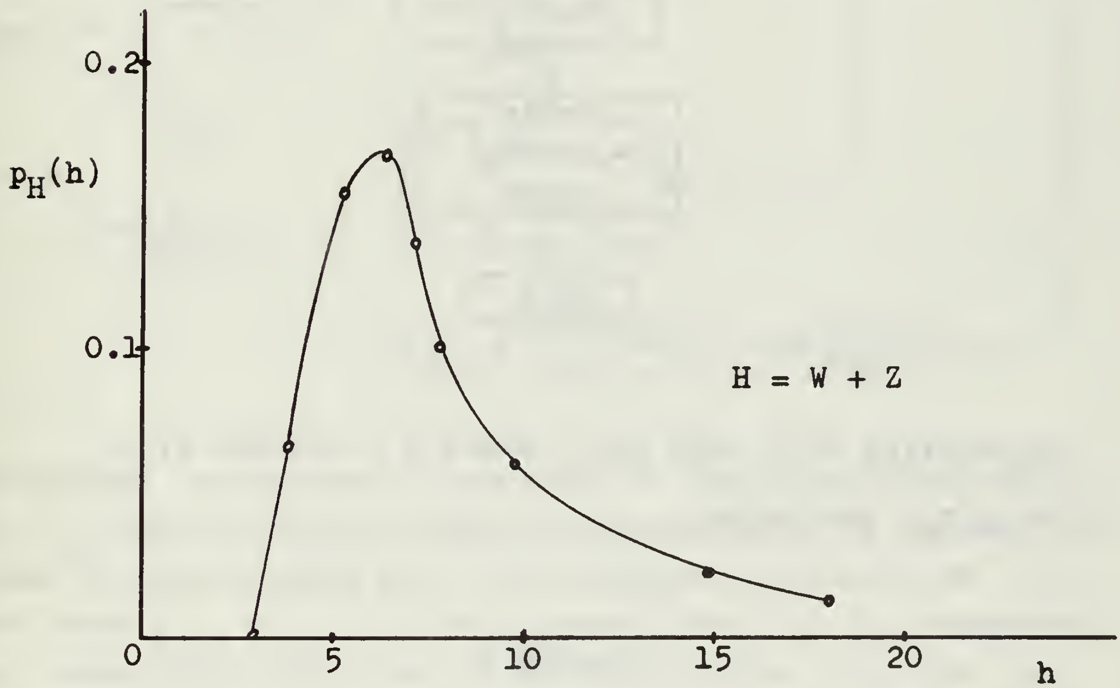
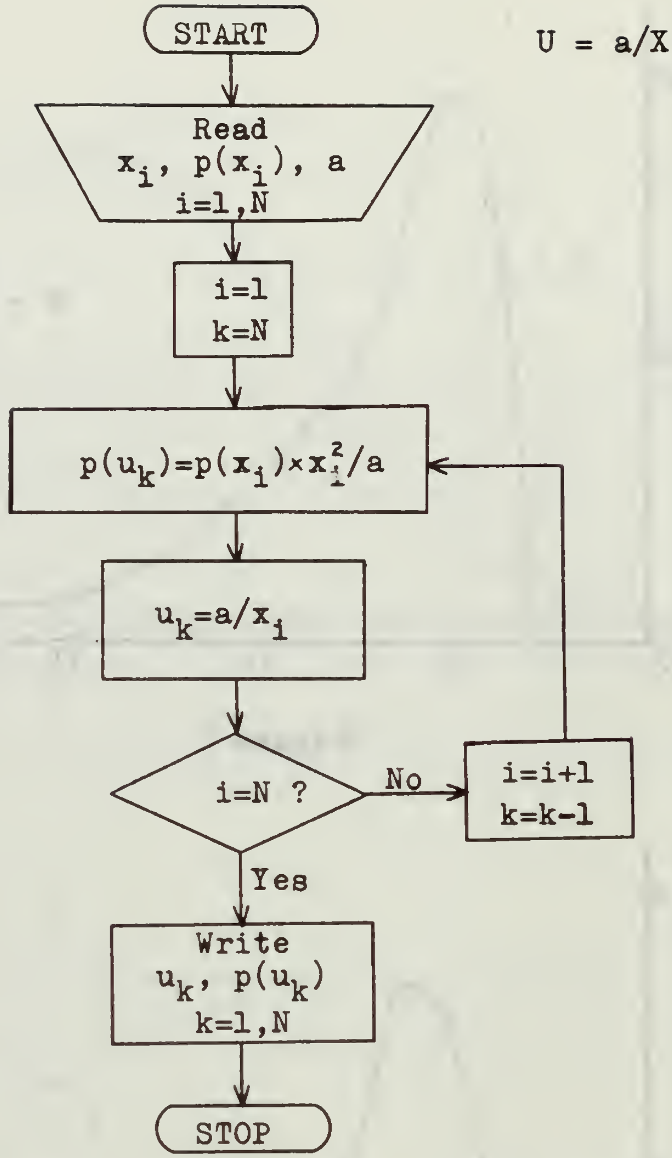
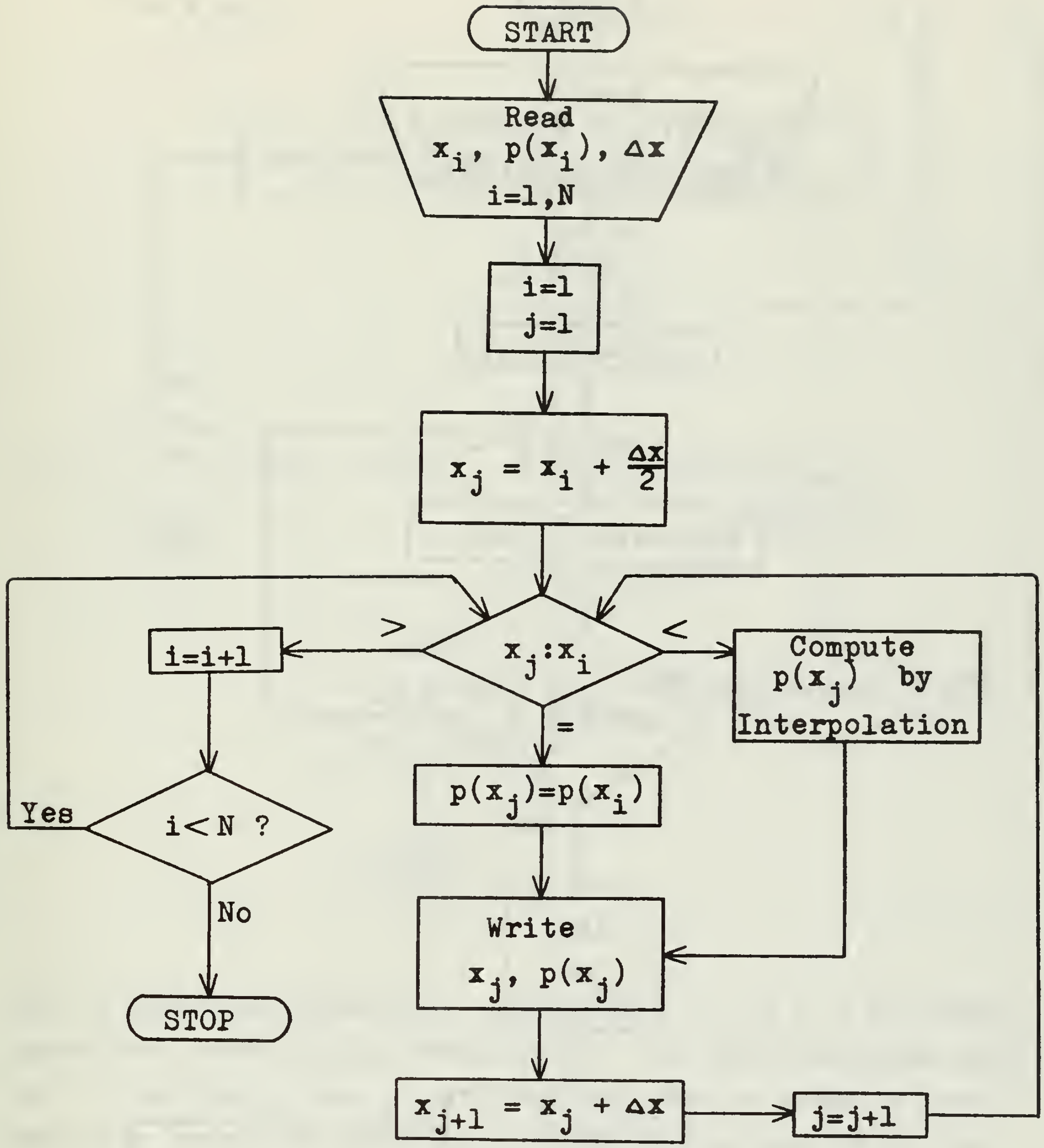


Figure 4



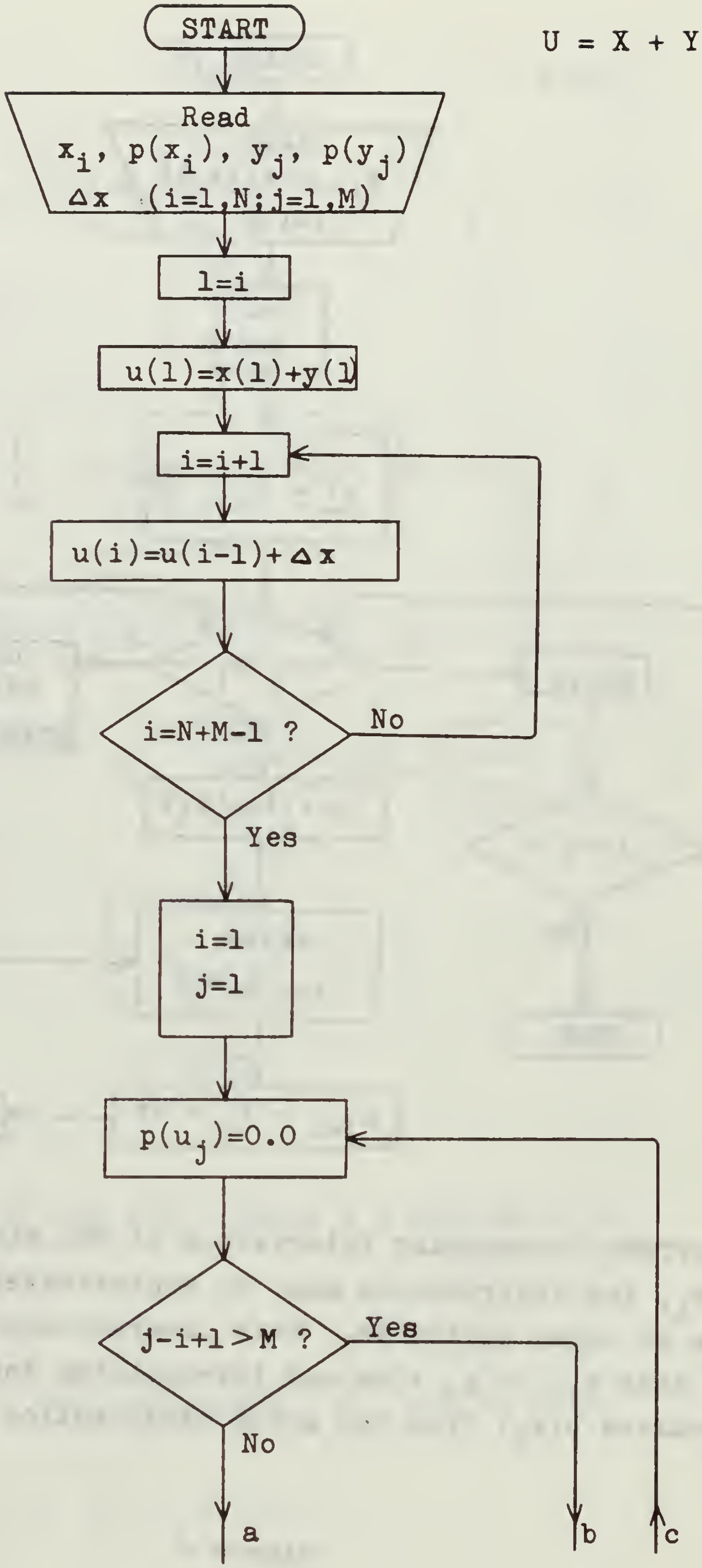
Calculation of u_k and $p(u_k)$ where $U = a/X$ and a, x_i, and $p(x_i)$ are known.

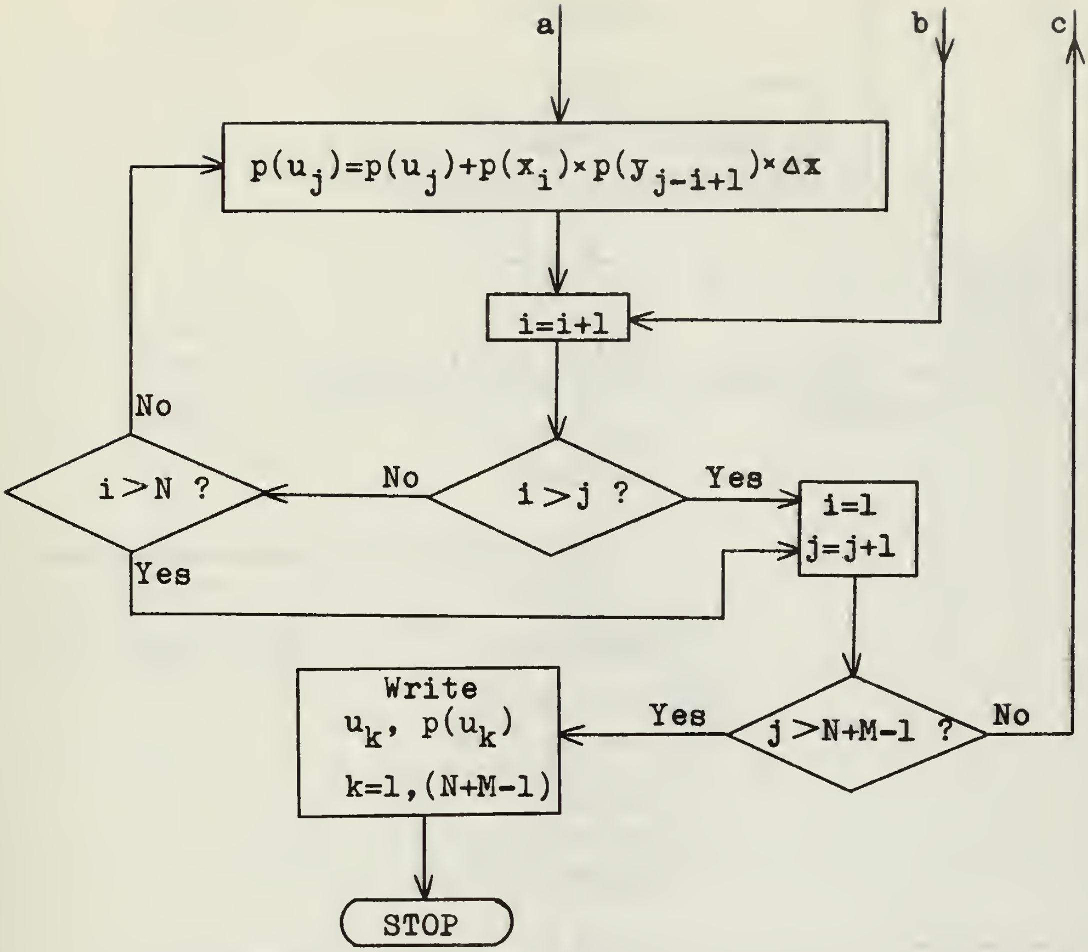
Figure 5



To perform rectangular integration of the distribution $p(x_i)$ vs. x_i, the distribution must be approximated by rectangular areas of equal widths Δx. This program calculates the x_j's such that $x_{j+1} = x_j + \Delta x$ and interpolates for the corresponding values $p(x_j)$ from the given distribution $p(x_i)$ vs. x_i.

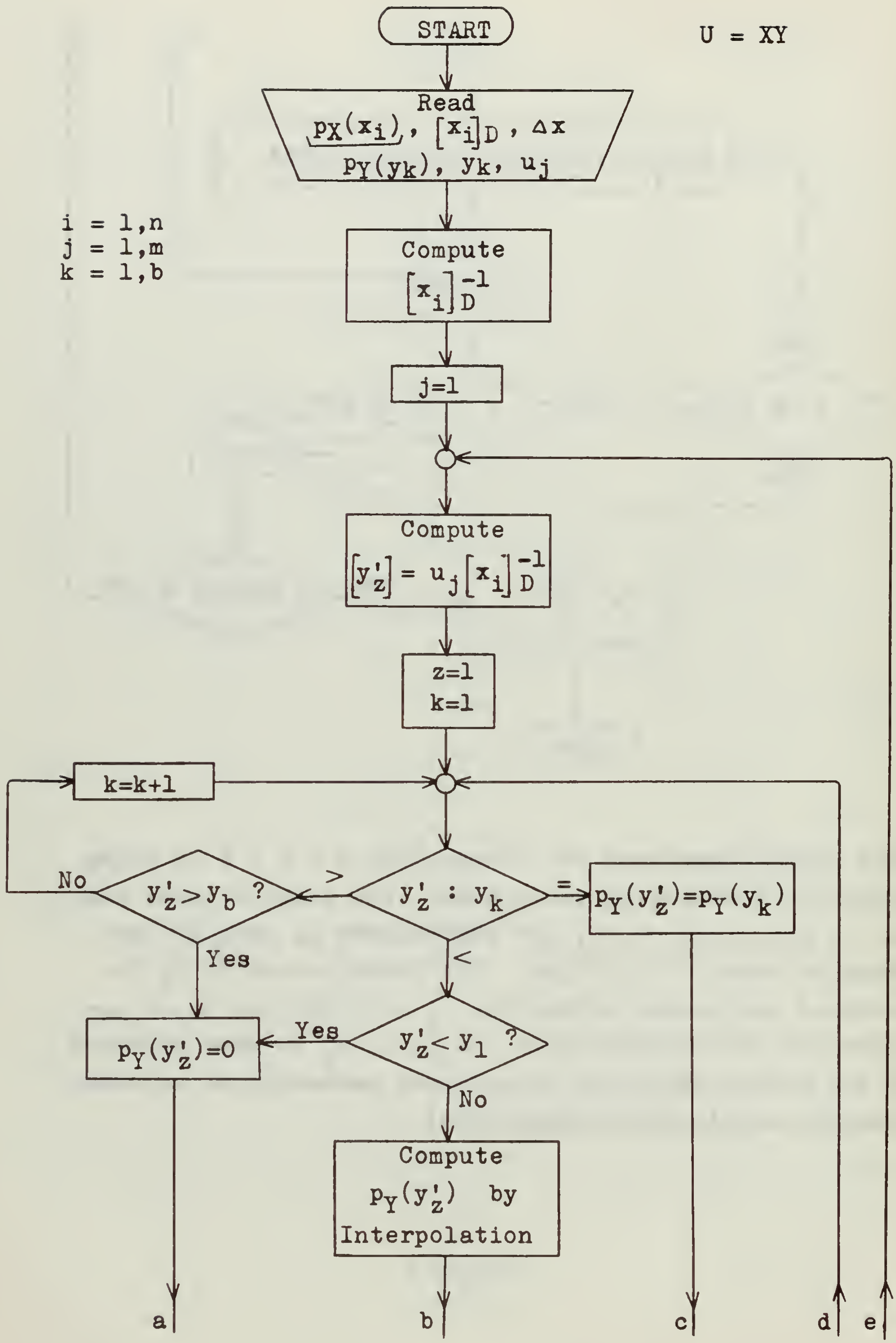
Figure 6


START
U = X + Y
Read
x_i, p(x_i), y_j, p(y_j)
Δx (i=1,N; j=1,M)
l=i
u(l)=x(l)+y(l)
i=i+1
u(i)=u(i-1)+Δx
i=N+M-1 ?
No
Yes
i=1
j=1
p(u_j)=0.0
j-i+1 > M ?
Yes
No
a
b
c



This program performs the convolution U = X + Y by using graphical convolution techniques. The distributions $p(x_i)$ vs. x_i and $p(y_j)$ vs. y_j are represented by sets of rectangular areas of width Δx. The distribution of x_i is reversed and passed across the y_j distribution in Δx increments. The distribution $p(u_k)$ vs. u_k is then obtained by the normal graphical convolution mathematical calculations [Bracewell 1965, Healy 1969].

Figure 7



START
U = XY
Read
$p_X(x_i)$, $[x_i]_D$, Δx
$p_Y(y_k)$, y_k, u_j
i = 1,n
j = 1,m
k = 1,b
Compute
$[x_i]_D^{-1}$
j=1
Compute
$[y'_z] = u_j [x_i]_D^{-1}$
z=1
k=1
k=k+1
No
$y'_z > y_b$?
>
$y'_z : y_k$
=
$p_Y(y'_z)=p_Y(y_k)$
Yes
<
$p_Y(y'_z)=0$
Yes
$y'_z < y_1$?
No
Compute
$p_Y(y'_z)$ by
Interpolation
a
b
c
d
e

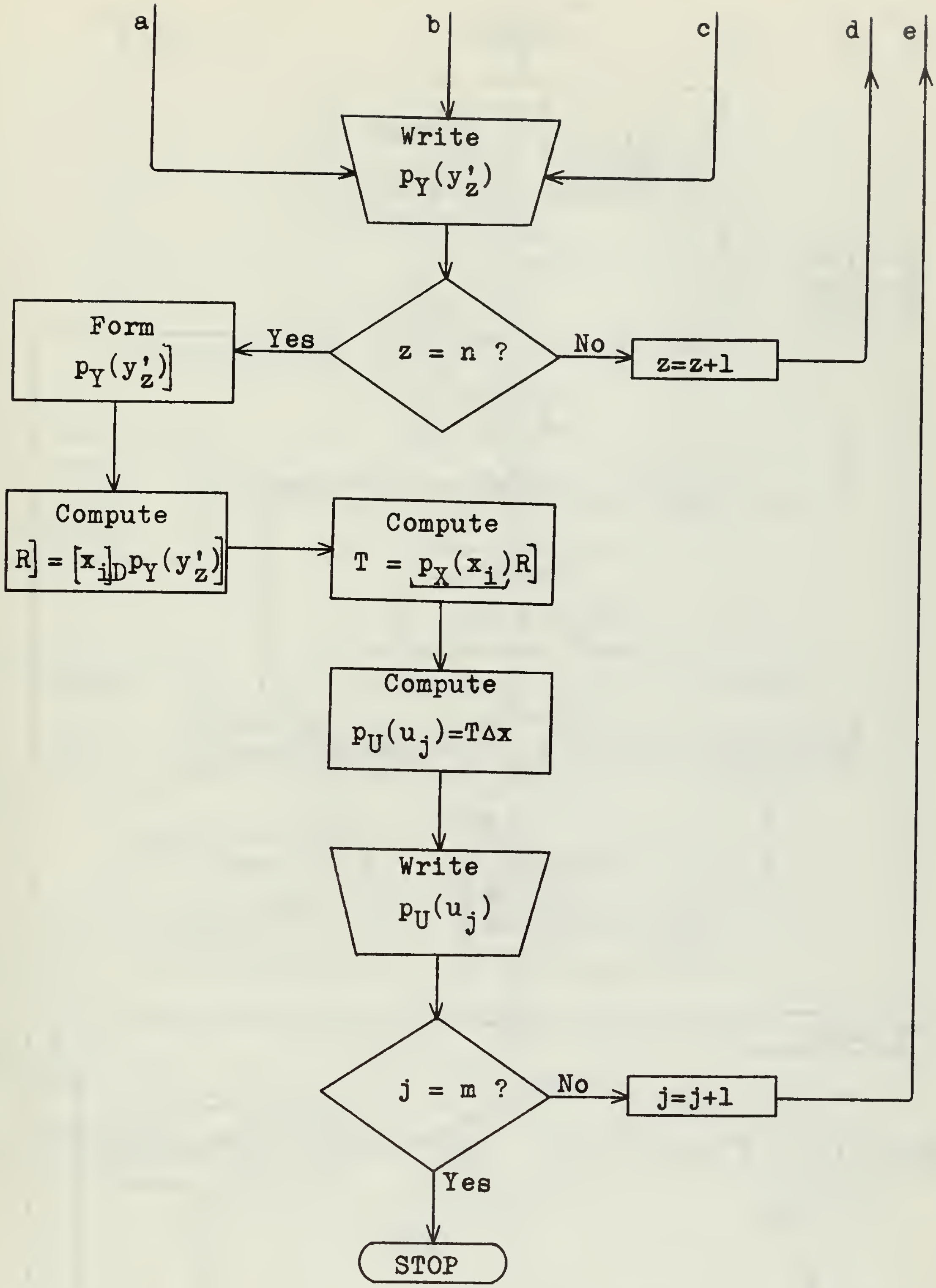
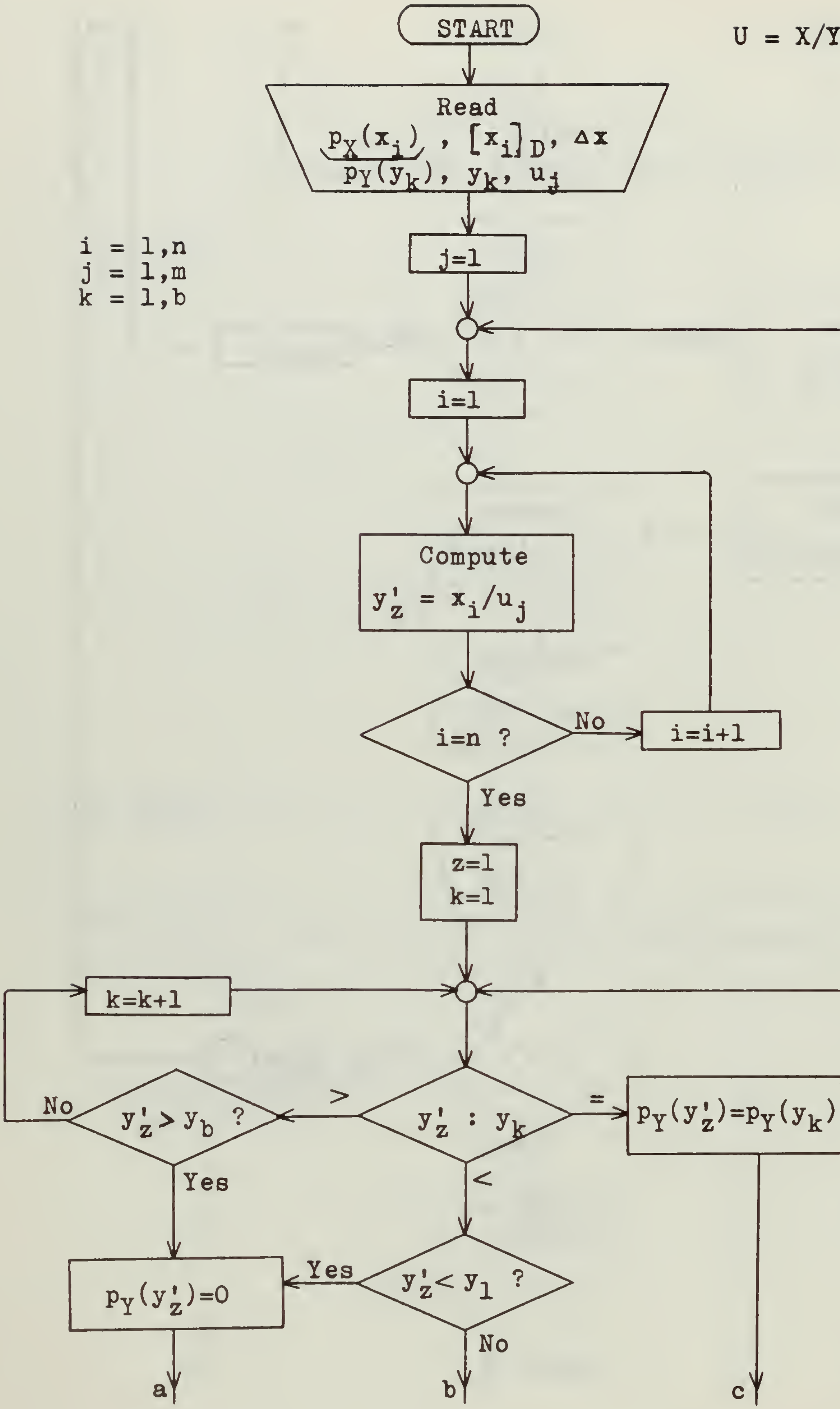

a
b
c
d
e
Write
$p_Y(y_z')$
$z = n$?
Yes
No
$z=z+1$
Form
$p_Y(y_z')]$
Compute
$R] = [x_i]_D p_Y(y_z')]$
Compute
$T = p_X(x_i)R]$
Compute
$p_U(u_j)=T\Delta x$
Write
$p_U(u_j)$
$j = m$?
No
$j=j+1$
Yes
STOP

Figure 8



U = X/Y
START
Read
$p_X(x_i)$, $[x_i]_D$, Δx
$p_Y(y_k)$, y_k, u_j
i = 1,n
j = 1,m
k = 1,b
j=1
i=1
Compute
$y'_z = x_i/u_j$
i=n ?
No
i=i+1
Yes
z=1
k=1
k=k+1
No
$y'_z > y_b$?
>
$y'_z : y_k$
=
$p_Y(y'_z)=p_Y(y_k)$
Yes
<
$p_Y(y'_z)=0$
Yes
$y'_z < y_1$?
No
a
b
c
d
e

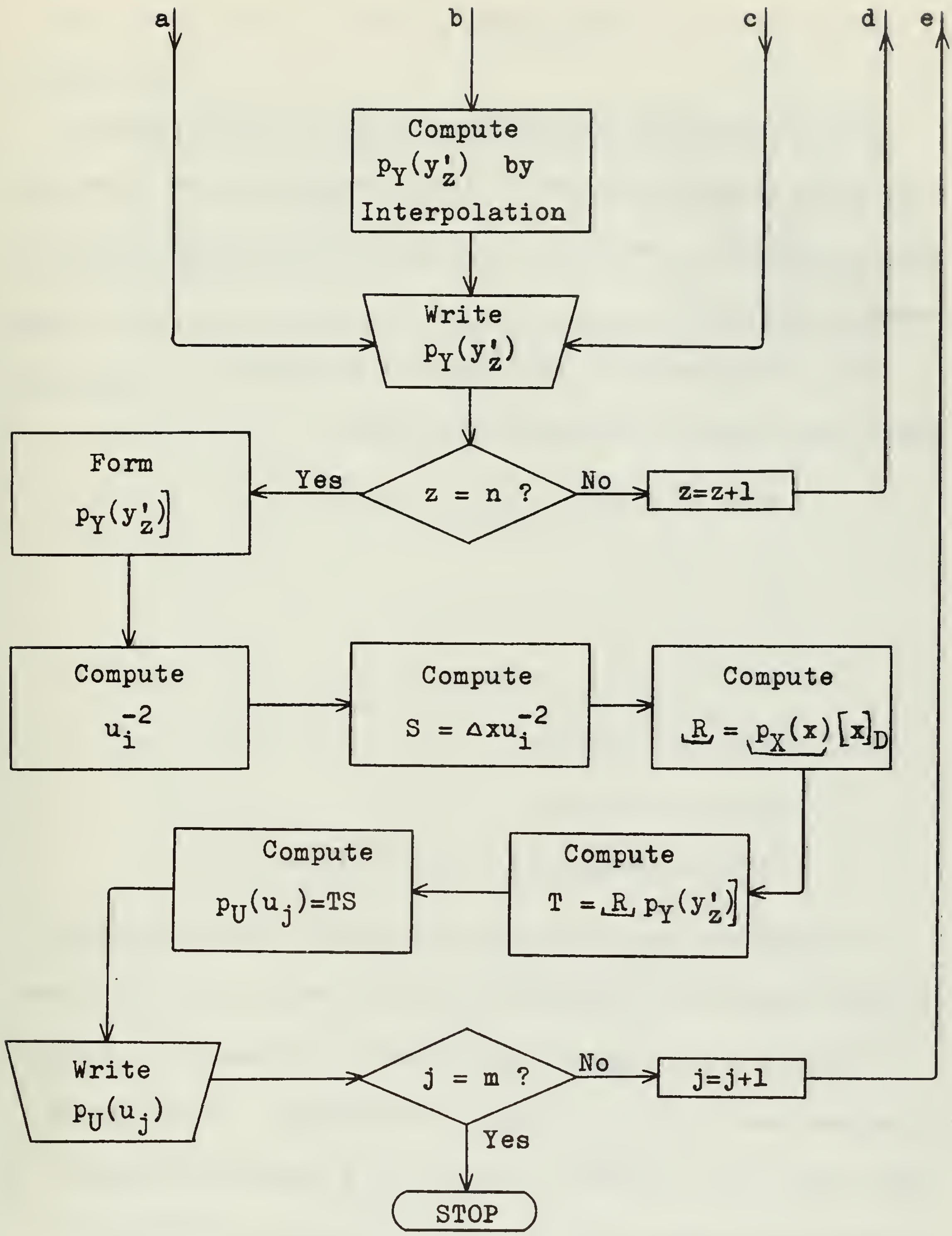

a
b
c
d
e
Compute
$p_Y(y'_z)$ by
Interpolation
Write
$p_Y(y'_z)$
Form
$p_Y(y'_z)]$
Yes
$z = n$?
No
$z=z+1$
Compute
u_i^{-2}
Compute
$S = \Delta x u_i^{-2}$
Compute
$R = p_X(x)[x]_D$
Compute
$p_U(u_j)=TS$
Compute
$T = R\, p_Y(y'_z)]$
Write
$p_U(u_j)$
$j = m$?
No
$j=j+1$
Yes
STOP

Figure 9

APPENDIX I

The solution to the deterministic problem has been obtained by using three separate programs. Linear interpolation and rectangular approximations have been used to perform all convolution and rectangular integration.

The problem and given information is as follows:

Obtain the cumulative distribution of θ where

$$\theta = C + \frac{A}{X(x)} + \frac{C}{Y(y)} + W(w)$$

$C = 0.175$

$A = 8.1$

$B = 27.8$

$X(x)$ is $N(130;10)$

$Y(y)$ is $N(65;12)$

$W(w)$ is uniform 0.875 from $-4/7$ to $+4/7$

Program one inverts the distribution x vs. p(x) and multiplies it by the constant A. Program two does the same for $B/Y(y)$. Since both $X(x)$ and $Y(y)$ are normal distributions, the input data for these programs was taken from standard normal tables. 25 data points were used in each, the median and each 1/4 standard deviation out to 3 standard deviations. The resulting distributions, $X'(x)$ and $Y'(y)$, are then used as input data for program three. Program three performs the convolution $U(u) = X'(x) + Y'(y)$, the addition $U'(u) = U(u) + C$,

and the convolution $\theta = U'(u) + W(w)$ to obtain the cumulative distribution of θ.

To obtain highly accurate results a Δx of 0.002 was used for convolution and rectangular integration. The over-all accuracy for this type of solution will depend upon how closely the rectangular approximations resemble the continuous distributions which they represent.

APPENDIX I - PROGRAM ONE

```
C       EVALUATION OF THE PROBABILITY DENSITY FUNCTION, P(X'),
C       WHERE X'=A/X AND A, X, AND P(X) ARE KNOWN. P(X) HAS
C       NORMAL DISTRIBUTION. N=NUMBER OF POINTS X(I), M IS THE
C       STANDARD DEVIATION OF THE DISTRIBUTION P(X).

        DIMENSION X(50),PX(50),U(50),PU(50)
        READ(5,100)N
        READ(5,101)M
        READ(5,102)A
   100  FORMAT(I2)
   101  FORMAT(I2)
   102  FORMAT(F10.0)
        WRITE(6,210) N
   210  FORMAT(1H1,T20,'N = ',I2//)
        WRITE(6,211) M
   211  FORMAT(T20,'M = ',I2//)
        WRITE(6,212) A
   212  FORMAT(T20,'A = ',F10.5//)
        WRITE(6,213)
   213  FORMAT(T20,'X(I)',T50,'PX(I)'//)
        DO 300 I=1,N
        READ(5,103) X(I),PX(I)
   103  FORMAT(2F10.0)
        WRITE(6,214) X(I),PX(I)
   214  FORMAT(2E30.8)
   300  CONTINUE
        DO 301 I=1,N
        K=N-I+1
        PU(K)=(X(I)**2/A)*(PX(I)/M)
        U(K)=A/X(I)
   301  CONTINUE
        WRITE(6,200)
   200  FORMAT(//,T20,'X',T50,'P(X )',//)
        DO 302 I=1,N
   201  FORMAT(2E30.8)
        STOP
        END

        N = 25
        M = 10
        A = 8.1
```

X'	PX'
0.50624996E-01	0.13905164E 01
0.51428568E-01	0.27868738E 01
0.52258059E-01	0.51905861E 01
0.53114749E-01	0.91015186E 01
0.53999994E-01	0.14999995E 02
0.54915249E-01	0.23179779E 02
0.55862062E-01	0.33614029E 02
0.56842100E-01	0.45776794E 02
0.57857137E-01	0.53558014E 02
0.58909085E-01	0.70279877E 02
0.59999995E-01	0.79222473E 02
0.61132070E-01	0.83814819E 02
0.62307686E-01	0.83227264E 02
0.63529372E-01	0.77608536E 02
0.64799964E-01	0.67920517E 02
0.66122413E-01	0.55782471E 02
0.67499995E-01	0.43022202E 02
0.68936110E-01	0.31123703E 02
0.70434749E-01	0.21143661E 02
0.71999967E-01	0.13484363E 02
0.73636353E-01	0.80666637E 01
0.75348794E-01	0.45226297E 01
0.77142835E-01	0.23819437E 01
0.79024374E-01	0.11803312E 01
0.80999970E-01	0.54320967E 00

APPENDIX I - PROGRAM TWO

```
C     EVALUATION OF THE PROBABILITY DENSITY FUNCTICN, P(Y'),
C     WHERE Y'=B/Y AND B, Y, AND P(Y) ARE KNOWN. P(Y) HAS
C     NORMAL DISTRIBUTION. N=NUMBER OF POINTS Y(I), M IS THE
C     STANDARD DEVIATION OF THE DISTRIBUTION P(Y).
      DIMENSION X(50),PX(50),U(50),PU(50)
      READ(5,100)N
      READ(5,101)M
      READ(5,102)A
  100 FORMAT(I2)
  101 FORMAT(I2)
  102 FORMAT(F10.0)
      WRITE(6,210) N
  210 FORMAT(1H1,T20,'N = ',I2//)
      WRITE(6,211) M
  211 FORMAT(T20,'M = ',I2//)
      WRITE(6,212) A
  212 FORMAT(T20,'A = ',F10.5//)
      WRITE(6,213)
  213 FORMAT(T20,'Y(I)',T50,'PY(I)'//)
      DO 300 I=1,N
      READ(5,103) X(I),PX(I)
  103 FORMAT(2F10.0)
      WRITE(6,214) X(I),PX(I)
  214 FORMAT(2E30.8)
  300 CONTINUE
      DO 301 I=1,N
      K=N-I+1
      PU(K)=(X(I)**2/A)*(PX(I)/M)
      U(K)=A/X(I)
  301 CONTINUE
      WRITE(6,200)
  200 FORMAT(//,T20,'Y',T50,'P(Y )',//)
      DO 302 I=1,N
  201 FORMAT(2E30.8)
      STOP
      END
```

N = 25
M = 12
A = 27.8

Y'	PY'
0.27524740E 00	0.13454551E 00
0.28367329E 00	0.26197964E 00
0.29263145E 00	0.47343379E 00
0.30217373E 00	0.80428284E 00
0.31235939E 00	0.12821751E 01
0.32325566E 00	0.19132919E 01
0.33493960E 00	0.26742373E 01
0.34749985E 00	0.35031176E 01
0.36103874E 00	0.43010139E 01
0.37567550E 00	0.49425173E 01
0.39154911E 00	0.53205528E 01
0.40882331E 00	0.53600159E 01
0.42769212E 00	0.50520163E 01
0.44838685E 00	0.44558611E 01
0.47118622E 00	0.36740417E 01
0.49642831E 00	0.28304844E 01
0.52452803E 00	0.20377035E 01
0.55599976E 00	0.13684053E 01
0.59148908E 00	0.85751051E 00
0.63181788E 00	0.50082946E 00
0.67804843E 00	0.27210426E 00
0.73157859E 00	0.13721460E 00
0.79428536E 00	0.64261079E-01
0.86874962E 00	0.27932845E-01
0.95862025E 00	0.11092324E-01

```
      CALCULATION OF U=X+Y WHERE X, P(X), Y, P(Y), ARE GIVEN
      AND THE GIVEN VALUES OF X(I) AND Y(I)  ARE INCREMENTED
      BY DX.

      DIMENSION X(500),PX(500),Y(500),PY(500),U(500),PU(500)
      PW(600),Z(1000),PZ(1000),F(500),W(600)
      READ(5,101) DX
  101 FORMAT(F10.0)
      WRITE(6,210) DX
  210 FORMAT(1H1,T20,'DX =',F10.3//)
      READ(5,106) C
  106 FORMAT(F10.0)
      WRITE(6,213) C
  213 FORMAT(T20,'C =',F10.8//)
      READ(5,105) N
  105 FORMAT(I5)
      WRITE(6,211) N
  211 FORMAT(T20,'N =',I5//)
      READ(5,104) M
  104 FORMAT(I5)
      WRITE(6,212) M
  212 FORMAT(//,T20,'M =',I5//)
      WRITE(6,202)
  202 FORMAT(//,T20,'X(I)',T50,'PX(I)'//)
      CALL ONE(X,PX)
      WRITE(6,203)
  203 FORMAT(//,T20,'Y(I)',T50,'PY(I)'//)
      CALL ONE(Y,PY)
      K=N+M-1
      U(1)=X(1)+Y(1)
      DO 302 I=2,K
  302 U(I)=U(I-1)+DX
      DO 303 J=1,K
      PU(J)=0.0
      DO 304 I=1,J
      IF((J-I+1).GT.M) GO TO 304
      IF(I.GT.N) GO TO 303
      PU(J)=PU(J)+PX(I)*PY(J-I+1)*DX
  304 CONTINUE
  303 CONTINUE

C     CALCULATION OF U(J)=U(J)+C WHERE 'C' IS A CONSTANT

      DO 306 I=1,K
      U(I)=U(I)+C
  306 CONTINUE
      WRITE(6,200)
  200 FORMAT(1H1,T20,'U(I)',T50,'PU(I)'//)
      DO 305 I=1,K
  305 WRITE(6,201) U(I),PU(I)
  201 FORMAT(2E30.8)

C     CALCULATION AND PLOT OF CUMULATIVE DISTRIBUTION

      F(1)=PU(1)*DX
      DO 320 I=2,K
      F(I)=F(I-1)+PU(I)*DX
  320 CONTINUE
      WRITE(6,225)
  225 FORMAT(1H1,T20,'U(J)',T40,'CUMULATIVE DISTRIBUTION')
      DO 321 I=1,K
      WRITE(6,201) U(I),F(I)
  321 CONTINUE

      PERFORM THE CONVOLUTION OF 'U' WITH THE CONSTANT PDF
      'W' (Z=U+W).

      CALL TWO(W,PW,L)
      NM=K+L-1
```

```
      Z(1)=U(1)+W(1)
      DO 310 I=2,NM
  310 Z(I)=Z(I-1)+DX
      DO 311 J=1,NM
      PZ(J)=0.0
      DO 312 I=1,J
      IF((J-I+1).GT.L) GO TO 312
      IF(I.GT.K) GO TO 311
      PZ(J)=PZ(J)+PU(I)*PW(J-I+1)*DX
  312 CONTINUE
  311 CONTINUE
      WRITE(6,230)
  230 FORMAT(1H1,T20,'Z(I)',T50,'PZ(I)'//)
      DO 313 I=1,NM
  313 WRITE(6,231) Z(I),PZ(I)
  231 FORMAT(2E30.8)
C     CALCULATION OF THE CUMULATIVE DISTRIBUTION OF 'Z'.
      PZ(1)=PZ(1)*DX
      DO 330 I=2,NM
      PZ(I)=PZ(I-1)+PZ(I)*DX
  330 CONTINUE
      WRITE(6,220)
  220 FORMAT(1H1,T20,'U(I)',T40,'CUMULATIVE DISTRIBUTION')
      DO 331 I=1,NM
  221 FORMAT(2E30.8)
  331 CONTINUE
      STOP
      END

      SUBROUTINE ONE(XE,PXE)
C     FROM GIVEN VALUES OF X(I) AND PX(I),(THESE ARE U(I)
      AND PU(I) PREVIOUSLY CALCULATED), AND A CHOSEN DX, WE
      OBTAIN THE VALUES OF X(J), WITH INTEVALS DX, AND
      CALCULATE THE CORRESPONDING VALUES OF PX(J) BY LINEAR
      INTERPOLATION.
      DIMENSION X(500),PX(500),XE(500),PXE(500)
      READ(5,100) N
  100 FORMAT(I2)
      READ(5,101) DX
  101 FORMAT(F10.0)
      DO 300 I=1,N
  300 READ(5,102) X(I), PX(I)
  102 FORMAT(2F10.0)
      I=1
      J=1
      XE(J)=X(I)+DX/2
  400 IF(XE(J)-X(I)) 500,501,502
  502 I=I+1
      IF(I.LT.(N+1)) GO TO 400
      GO TO 203
  501 PXE(J)=PX(I)
      GO TO 200
  500 PXE(J)=PX(I-1)+(PX(I)-PX(I-1))/(X(I)-X(I-1))*(XE(J)-
      X(I-1))
  200 WRITE(6,201)  XE(J), PXE(J)
  201 FORMAT(2E30.8)
      XE(J+1)=XE(J)+DX
      J=J+1
      GO TO 400
  203 RETURN
      END

      SUBROUTINE TWO(W,PW,L)
C     THIS PROGRAM TAKES THE GIVEN CONSTANT PROBABILITY
C     FUNCTION 'W', WITH AMPLITUDE 0.875 FROM -4/7 TO +4/7,
```

```
      AND OBTAINS THE VALUES OF W(I) WITH INCREMENT DX=0.002
      DIMENSION W(600), PW(600)
      W(1)=-0.571
      PW(1)=0.875
      DX=0.002
      L=571
      WRITE(6,250)
  250 FORMAT(1H1,T20,'W(I)',T50,'PW(I)'//)
      DO 350 I=2,L
      W(I)=W(I-1)+DX
      PW(I)=PW(I-1)
  350 CONTINUE
      DO 351 I=1,L
      WRITE(6,251) W(I),PW(I)
  251 FORMAT(2E30.8)
  351 CONTINUE
      RETURN
      END

      N = 15
      M = 342
      C = 0.175
      DX = 0.002
```

THETA	CUMULATIVE DISTRIBUTION
-0.68128109E-01	0.35162280E-08
-0.66128075E-01	0.24354559E-07
-0.64128041E-01	0.93342241E-07
-0.62128045E-01	0.29366913E-06
-0.60128044E-01	0.71189930E-06
-0.58128044E-01	0.14791967E-05
-0.56128044E-01	0.27345241E-05
-0.54128043E-01	0.46203268E-05
-0.52128043E-01	0.72799021E-05
-0.50128043E-01	0.10858982E-04
-0.48128042E-01	0.15508529E-04
-0.46128042E-01	0.21388027E-04
-0.44128042E-01	0.28669005E-04
-0.42128041E-01	0.37537306E-04
-0.40128041E-01	0.48194212E-04
-0.38128041E-01	0.60856095E-04
-0.36128040E-01	0.75755495E-04
-0.34128040E-01	0.93142065E-04
-0.32128040E-01	0.11328337E-03
-0.30128039E-01	0.13646450E-03
-0.28128039E-01	0.16298697E-03
-0.26128039E-01	0.19316789E-03
-0.24128038E-01	0.22734021E-03
-0.22128038E-01	0.26585301E-03
-0.20128038E-01	0.30907127E-03
-0.18128037E-01	0.35737362E-03
-0.16128037E-01	0.41115121E-03
-0.14128037E-01	0.47080661E-03
-0.12128036E-01	0.53675333E-03
-0.10128036E-01	0.60941558E-03
-0.81280358E-02	0.68922667E-03
-0.61280355E-02	0.77662780E-03
-0.41280352E-02	0.87206601E-03
-0.21280353E-02	0.97599253E-03
-0.12803543E-03	0.10888616E-02
0.18719644E-02	0.12111294E-02
0.38719643E-02	0.13432535E-02
0.58719628E-02	0.14856923E-02
0.78719594E-02	0.16389030E-02
0.98719560E-02	0.18033390E-02
0.11871953E-01	0.19794477E-02
0.13871949E-01	0.21676687E-02
0.15871946E-01	0.23684341E-02
0.17871942E-01	0.25821684E-02
0.19871939E-01	0.28092891E-02
0.21871936E-01	0.30502048E-02
0.23871932E-01	0.33053122E-02
0.25871929E-01	0.35749928E-02
0.27871925E-01	0.38596098E-02
0.29871922E-01	0.41595064E-02
0.31871919E-01	0.44750199E-02
0.33871915E-01	0.48064664E-02
0.35871912E-01	0.51541552E-02
0.37871908E-01	0.55183806E-02
0.39871905E-01	0.58994144E-02
0.41871902E-01	0.62975064E-02
0.43871898E-01	0.67128837E-02
0.45871895E-01	0.71457513E-02
0.47871891E-01	0.75962991E-02
0.49871888E-01	0.80647096E-02
0.51871885E-01	0.85511431E-02
0.53871881E-01	0.90557523E-02
0.55871878E-01	0.95786639E-02
0.57871874E-01	0.10119986E-01
0.59871871E-01	0.10679796E-01
0.61871868E-01	0.11258151E-01
0.63871861E-01	0.11855092E-01
0.65871835E-01	0.12470651E-01
0.67871809E-01	0.13104849E-01

0.69871783E-01	0.13757691E-01
0.71871758E-01	0.14429174E-01
0.73871732E-01	0.15119277E-01
0.75871706E-01	0.15827950E-01
0.77871680E-01	0.16555130E-01
0.79871655E-01	0.17300732E-01
0.81871629E-01	0.18064659E-01
0.83871603E-01	0.18846810E-01
0.85871577E-01	0.19647073E-01
0.87871552E-01	0.20465329E-01
0.89871526E-01	0.21301460E-01
0.91871500E-01	0.22155326E-01
0.93871474E-01	0.23026783E-01
0.95871449E-01	0.23915663E-01
0.97871423E-01	0.24821792E-01
0.99871397E-01	0.25744982E-01
0.10187137E 00	0.26685044E-01
0.10387135E 00	0.27641781E-01
0.10587132E 00	0.28614990E-01
0.10787129E 00	0.29604476E-01
0.10987127E 00	0.30610032E-01
0.11187124E 00	0.31631447E-01
0.11387122E 00	0.32668509E-01
0.11587119E 00	0.33720996E-01
0.11787117E 00	0.34788683E-01
0.11987114E 00	0.35871338E-01
0.12187111E 00	0.36968727E-01
0.12387109E 00	0.38080610E-01
0.12587106E 00	0.39206754E-01
0.12787104E 00	0.40346917E-01
0.12987101E 00	0.41500859E-01
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0.10218582E 01	0.79616970E 00
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0.10298576E 01	0.80314630E 00
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0.10578556E 01	0.82756442E 00
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0.10618553E 01	0.83105272E 00
0.10638552E 01	0.83279687E 00
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0.10718546E 01	0.83977348E 00
0.10738544E 01	0.84151763E 00
0.10758543E 01	0.84326178E 00

0.10778542E 01	0.84500587E 00
0.10798540E 01	0.84674984E 00
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0.10838537E 01	0.85023695E 00
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0.10938530E 01	0.85894376E 00
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0.10998526E 01	0.86415422E 00
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0.11098518E 01	0.87279928E 00
0.11118517E 01	0.87452024E 00
0.11138515E 01	0.87623787E 00
0.11158514E 01	0.87795186E 00
0.11178513E 01	0.87966186E 00
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0.11258507E 01	0.88645470E 00
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0.11938457E 01	0.93761635E 00
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0.12178440E 01	0.95126945E 00
0.12198439E 01	0.95228642E 00

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0.12258434E 01	0.95522726E 00
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0.12318430E 01	0.95800507E 00
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0.12358427E 01	0.95976812E 00
0.12378426E 01	0.96062338E 00
0.12398424E 01	0.96146142E 00
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0.12438421E 01	0.96308655E 00
0.12458420E 01	0.96387398E 00
0.12478418E 01	0.96464491E 00
0.12498417E 01	0.96539956E 00
0.12518415E 01	0.96613818E 00
0.12538414E 01	0.96686095E 00
0.12558413E 01	0.96756804E 00
0.12578411E 01	0.96825969E 00
0.12598410E 01	0.96893615E 00
0.12618408E 01	0.96959764E 00
0.12638407E 01	0.97024441E 00
0.12658405E 01	0.97087669E 00
0.12678404E 01	0.97149473E 00
0.12698402E 01	0.97209877E 00
0.12718401E 01	0.97268897E 00
0.12738400E 01	0.97326565E 00
0.12758398E 01	0.97382903E 00
0.12778397E 01	0.97437936E 00
0.12798395E 01	0.97491688E 00
0.12818394E 01	0.97544175E 00
0.12838392E 01	0.97595429E 00
0.12858391E 01	0.97645468E 00
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0.12898388E 01	0.97742003E 00
0.12918386E 01	0.97788548E 00
0.12938385E 01	0.97833973E 00
0.12958384E 01	0.97878301E 00
0.12978382E 01	0.97921556E 00
0.12998381E 01	0.97963762E 00
0.13018379E 01	0.98004943E 00
0.13038378E 01	0.98045117E 00
0.13058376E 01	0.98084307E 00
0.13078375E 01	0.98122537E 00
0.13098373E 01	0.98159826E 00
0.13118372E 01	0.98196191E 00
0.13138371E 01	0.98231655E 00
0.13158369E 01	0.98266238E 00
0.13178368E 01	0.98299956E 00
0.13198366E 01	0.98332834E 00
0.13218365E 01	0.98364890E 00
0.13238363E 01	0.98396140E 00
0.13258362E 01	0.98426610E 00
0.13278360E 01	0.98456317E 00
0.13298359E 01	0.98485273E 00
0.13318357E 01	0.98513502E 00
0.13338356E 01	0.98541021E 00
0.13358355E 01	0.98567849E 00
0.13378353E 01	0.98593998E 00
0.13398352E 01	0.98619485E 00
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0.13458347E 01	0.98692119E 00
0.13478346E 01	0.98715109E 00
0.13498344E 01	0.98737508E 00
0.13518343E 01	0.98759335E 00
0.13538342E 01	0.98780602E 00
0.13558340E 01	0.98801327E 00
0.13578339E 01	0.98821527E 00
0.13598337E 01	0.98841208E 00
0.13618336E 01	0.98860395E 00
0.13638334E 01	0.98879093E 00

0.13658333E 01	0.98897314E 00
0.13678331E 01	0.98915070E 00
0.13698330E 01	0.98932374E 00
0.13718328E 01	0.98949236E 00
0.13738327E 01	0.98965669E 00
0.13758326E 01	0.98981676E 00
0.13778324E 01	0.98997277E 00
0.13798323E 01	0.99012476E 00
0.13818321E 01	0.99027282E 00
0.13838320E 01	0.99041706E 00
0.13858318E 01	0.99055761E 00
0.13878317E 01	0.99069452E 00
0.13898315E 01	0.99082792E 00
0.13918314E 01	0.99095792E 00
0.13938313E 01	0.99108464E 00
0.13958311E 01	0.99120814E 00
0.13978310E 01	0.99132848E 00
0.13998308E 01	0.99144578E 00
0.14018307E 01	0.99156016E 00
0.14038305E 01	0.99167162E 00
0.14058304E 01	0.99178028E 00
0.14078302E 01	0.99188614E 00
0.14098301E 01	0.99198931E 00
0.14118299E 01	0.99208987E 00
0.14138298E 01	0.99218786E 00
0.14158297E 01	0.99228334E 00
0.14178295E 01	0.99237639E 00
0.14198294E 01	0.99246705E 00
0.14218292E 01	0.99255538E 00
0.14238291E 01	0.99264145E 00
0.14258289E 01	0.99272531E 00
0.14278288E 01	0.99280703E 00
0.14298286E 01	0.99288666E 00
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0.14338284E 01	0.99303997E 00
0.14358282E 01	0.99311376E 00
0.14378281E 01	0.99318570E 00
0.14398279E 01	0.99325585E 00
0.14418278E 01	0.99332428E 00
0.14438276E 01	0.99339098E 00
0.14458275E 01	0.99345601E 00
0.14478273E 01	0.99351943E 00
0.14498272E 01	0.99358124E 00
0.14518270E 01	0.99364150E 00
0.14538269E 01	0.99370027E 00
0.14558268E 01	0.99375755E 00
0.14578266E 01	0.99381340E 00
0.14598265E 01	0.99386781E 00
0.14618263E 01	0.99392086E 00
0.14638262E 01	0.99397254E 00
0.14658260E 01	0.99402291E 00
0.14678259E 01	0.99407202E 00
0.14698257E 01	0.99411988E 00
0.14718256E 01	0.99416655E 00
0.14738255E 01	0.99421203E 00
0.14758253E 01	0.99425638E 00
0.14778252E 01	0.99429965E 00
0.14798250E 01	0.99434185E 00
0.14818249E 01	0.99438298E 00
0.14838247E 01	0.99442309E 00
0.14858246E 01	0.99446225E 00
0.14878244E 01	0.99450046E 00
0.14898243E 01	0.99453777E 00
0.14918242E 01	0.99457413E 00
0.14938240E 01	0.99460959E 00
0.14958239E 01	0.99464422E 00
0.14978237E 01	0.99467796E 00
0.14998236E 01	0.99471086E 00
0.15018234E 01	0.99474299E 00
0.15038233E 01	0.99477428E 00
0.15058231E 01	0.99480480E 00
0.15078230E 01	0.99483454E 00

0.15098228E 01	0.99486357E 00
0.15118227E 01	0.99489188E 00
0.15138226E 01	0.99491948E 00
0.15158224E 01	0.99494636E 00
0.15178223E 01	0.99497259E 00
0.15198221E 01	0.99499816E 00
0.15218220E 01	0.99502307E 00
0.15238218E 01	0.99504739E 00
0.15258217E 01	0.99507105E 00
0.15278215E 01	0.99509418E 00
0.15298214E 01	0.99511671E 00
0.15318213E 01	0.99513870E 00
0.15338211E 01	0.99516016E 00
0.15358210E 01	0.99518108E 00
0.15378208E 01	0.99520153E 00
0.15398207E 01	0.99522144E 00
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0.15438204E 01	0.99525982E 00
0.15458202E 01	0.99527836E 00
0.15478201E 01	0.99529642E 00
0.15498199E 01	0.99531406E 00
0.15518198E 01	0.99533129E 00
0.15538197E 01	0.99534810E 00
0.15558195E 01	0.99536449E 00
0.15578194E 01	0.99538046E 00
0.15598192E 01	0.99539602E 00
0.15618191E 01	0.99541116E 00
0.15638189E 01	0.99542594E 00
0.15658188E 01	0.99544036E 00
0.15678186E 01	0.99545443E 00
0.15698185E 01	0.99546814E 00
0.15718184E 01	0.99548149E 00
0.15738182E 01	0.99549448E 00
0.15758181E 01	0.99550712E 00
0.15778179E 01	0.99551946E 00
0.15798178E 01	0.99553150E 00
0.15818176E 01	0.99554318E 00
0.15838175E 01	0.99555457E 00
0.15858173E 01	0.99556565E 00
0.15878172E 01	0.99557650E 00
0.15898170E 01	0.99558705E 00
0.15918169E 01	0.99559730E 00
0.15938168E 01	0.99560732E 00
0.15958166E 01	0.99561709E 00
0.15978165E 01	0.99562663E 00
0.15998163E 01	0.99563593E 00
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0.16038160E 01	0.99565381E 00
0.16058159E 01	0.99566239E 00
0.16078157E 01	0.99567080E 00
0.16098156E 01	0.99567896E 00
0.16118155E 01	0.99568695E 00
0.16138153E 01	0.99569470E 00
0.16158152E 01	0.99570227E 00
0.16178150E 01	0.99570960E 00
0.16198149E 01	0.99571675E 00
0.16218147E 01	0.99572372E 00
0.16238146E 01	0.99573052E 00
0.16258144E 01	0.99573714E 00
0.16278143E 01	0.99574357E 00
0.16298141E 01	0.99574983E 00
0.16318140E 01	0.99575591E 00
0.16338139E 01	0.99576181E 00
0.16358137E 01	0.99576753E 00
0.16378136E 01	0.99577308E 00
0.16398134E 01	0.99577844E 00
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0.16438131E 01	0.99578875E 00
0.16458130E 01	0.99579370E 00
0.16478128E 01	0.99579847E 00
0.16498127E 01	0.99580312E 00
0.16518126E 01	0.99580759E 00

0.16538124E 01	0.99581194E 00
0.16558123E 01	0.99581617E 00
0.16578121E 01	0.99582028E 00
0.16598120E 01	0.99582428E 00
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0.16638117E 01	0.99583179E 00
0.16658115E 01	0.99583536E 00
0.16678114E 01	0.99583888E 00
0.16698112E 01	0.99584228E 00
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0.16738110E 01	0.99584872E 00
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0.16778107E 01	0.99585474E 00
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0.16838102E 01	0.99586308E 00
0.16858101E 01	0.99586564E 00
0.16878099E 01	0.99586815E 00
0.16898098E 01	0.99587053E 00
0.16918097E 01	0.99587286E 00
0.16938095E 01	0.99587512E 00
0.16958094E 01	0.99587727E 00
0.16978092E 01	0.99587935E 00
0.16998091E 01	0.99588132E 00
0.17018089E 01	0.99588323E 00
0.17038088E 01	0.99588507E 00
0.17058086E 01	0.99588680E 00
0.17078085E 01	0.99588847E 00
0.17098083E 01	0.99589008E 00
0.17118082E 01	0.99589157E 00
0.17138081E 01	0.99589300E 00
0.17158079E 01	0.99589437E 00
0.17178078E 01	0.99589568E 00
0.17198076E 01	0.99589694E 00
0.17218075E 01	0.99589813E 00
0.17238073E 01	0.99589920E 00
0.17258072E 01	0.99590021E 00
0.17278070E 01	0.99590117E 00
0.17298069E 01	0.99590206E 00
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0.17358065E 01	0.99590439E 00
0.17378063E 01	0.99590504E 00
0.17398062E 01	0.99590564E 00
0.17418060E 01	0.99590617E 00
0.17438059E 01	0.99590665E 00
0.17458057E 01	0.99590707E 00
0.17478056E 01	0.99590749E 00
0.17498055E 01	0.99590784E 00
0.17518053E 01	0.99590814E 00
0.17538052E 01	0.99590838E 00
0.17558050E 01	0.99590856E 00
0.17578049E 01	0.99590874E 00
0.17598047E 01	0.99590886E 00
0.17618046E 01	0.99590892E 00
0.17638044E 01	0.99590898E 00
0.17658043E 01	0.99590898E 00
0.17678041E 01	0.99590898E 00
0.17698040E 01	0.99590898E 00
0.17718039E 01	0.99590898E 00
0.17738037E 01	0.99590898E 00
0.17758036E 01	0.99590898E 00
0.17778034E 01	0.99590898E 00
0.17798033E 01	0.99590898E 00
0.17818031E 01	0.99590898E 00

APPENDIX II

The tabulated sample data for the non-deterministic case, Table II, was obtained from a modified form of equation (19). Using the modified equation,

$$\phi = 0.175 + \frac{27.8}{Y(y)} + \frac{8.1}{X(x)}$$

where $X(x)$ is $N(130;10)$

$Y(y)$ is $N(65;12)$

values of ϕ and sensitivity were calculated for chosen values of x and y. Assuming these to be measured values, the data was then used to obtain the cumulative distribution of ϕ.

Since the data indicates ϕ is more sensitive to changes in y than to changes in x, the problem is worked with x as a constant and y as a single variable as discussed in section III. Aitkin-Hermite interpolation [Hildebrand 1956] is used to estimate the values of y between data points. This interpolation scheme utilizes the sensitivity data and obtains a more accurate estimate than would linear interpolation. The calculation of the cumulative distribution of ϕ is then made using equation (35) in discrete form. As in the deterministic case, a Δx of 0.002 was used for rectangular integration.

APPENDIX II - PROGRAM ONE

```
C       CALCULATION OF THE CUMULATIVE DISTRIBUTION OF AN
        OUTPUT,  PHI  , FROM MEASURED VALUES OF THE PARAMETERS
        X, Y, AND THE SENSITIVITY OF  PHI  WITH RESPECT TO X
        AND Y. THE PROBABILITY DISTRIBUTIONS OF X AND Y ARE
        KNOWN, N(130;10) AND N(65;12) RESPECTIVELY, AND THE
        DATA POINTS HAVE BEEN SELECTED IN DX INTERVALS TO
        FACILITATE THE CALCULATIONS.

        DIMENSION ARG1(5,5),VAL(10),ARG(5),XC(5),F(5,2),Z(5),
        PYT(25),PXC(5),X1(200),PX(200),YT(25)

C       READ THE MEASURED VALUES OF X AND THEIR CORRESPONDING
C       PROBABILITIES, DENOTED BY XC AND PXC.

        READ(5,102) (XC(I),PXC(I),I=1,5)
        WRITE(6,103)
  103   FORMAT(//,T21,'XC',T47,'PXC',/)
        WRITE(6,106) (XC(I),PXC(I),I=1,5)
  106   FORMAT(2F26.5)
  101   FORMAT(F10.5)

        READ THE MEASURED VALUES OF  PHI  , DENOTED BY ARG1.
        THERE IS A POINT  PHI  CORRESPONDING TO EACH SET OF
        X - Y POINTS.

        READ(5,101) ((ARG1(I,J),J=1,5),I=1,5)

C       READ THE MEASURED VALUES OF Y AND THE CORRESPONDING
        SENSITIVITY OF  PHI  WITH RESPECT TO Y FOR EACH OF
        THESE VALUES, DENOTED BY F(I,J).

        READ(5,102) ((F(I,J),J=1,2),I=1,5)
  102   FORMAT(2F10.5)

        READ THE KNOWN CUMULATIVE DISTRIBUTION OF Y, DENOTED
        BY YT AND PYT.

        READ(5,102) (YT(I),PYT(I),I=1,25)
        X1(1)=0.5708710
        DO 204 K=1,168
        PT=0.0

C       X DENOTES THE VALUE OF  PHI  FOR WHICH THE CUMULATIVE
C       DISTRIBUTION IS BEING CALCULATED.

        X=X1(K)
        DO 202 I=1,5
        DO 201 J=1,5

        Z(J) DENOTES THE ARRAY OF ARG1 VALUES CORRESPONDING
        TO A PARTICULAR VALUE OF XC.

  201   Z(J)=ARG1(I,J)
        DX=10.0
        EPS=0.001
        NDIM=5

C       SUBROUTINE ATSM ORDERS THE ARRAY Z(J) SUCH THAT
        ABS(ARG(I)-X).GE.ABS(ARG(J)-X) IF I.GEJ. THIS IS
        REQUIRED FOR SUBROUTINE ONE.

        CALL ATSM(X,Z,F,5,2,ARG,VAL,NDIM)

C       SUBROUTINE ONE INTERPOLATES FUNCTION VALUE Y FOR A
        GIVEN ARGUMENT VALUE X USING THE TABLE (ARG,VAL) OF
        ARGUMENT, FUNCTION, AND DERIVATIVE VALUES FROM THE
C       SUBROUTINE ATSM. THIS SUBROUTINE USES THE AITKEN-
        HERMITE INTERPOLATION.
```

```
      CALL ONE(X,ARG,VAL,Y,NDIM,EPS,IER)
C     SUBROUTINE TWO USES LINEAR INTERPOLATION TO OBTAIN
      P(Y) FOR THE VALUE OF Y OBTAINED IN SUBROUTINE ONE.
      CALL TWO(Y,PY,YT,PYT)
      PT=PT+(1-PY)*PXC(I)
      PX(K)=PT*DX
  302 FORMAT(2E30.8)
  202 CONTINUE
      X1(K+1)=X1(K)+0.0020000
  204 CONTINUE
      WRITE(6,107)
  107 FORMAT(1H1,T20,'U(I)',T42,'CUMULATIVE DISTRIBUTION',//
      WRITE(6,302) (X1(K),PX(K),K=1,168)
      STOP
      END

      SUBROUTINE ATSM(X,Z,F,IROW,ICOL,ARG,VAL,NDIM)
      DIMENSION Z(1),F(1),ARG(1),VAL(1)
C     CASE IROW=1 IS CHECKED OUT
      IF(IROW-1)23,21,1
    1 N=NDIM
C     IF N IS GREATER THAN IROW, N IS SET EQUAL TO IROW.
      IF(N-IROW)3,3,2
    2 N=IROW
C     CASE IROW .GE. 2
C     SEARCHING FOR SUBSCRIPT J SUCH THAT Z(J) IS NEXT TO X.
    3 IF(Z(IROW)-Z(1))5,4,4
    4 J=IROW
      I=1
      GO TO 6
    5 I=IROW
      J=1
    6 K=(J+I)/2
      IF(X-Z(K))7,7,8
    7 J=K
      GO TO 9
    8 I=K
    9 IF(IABS(J-I)-1)10,10,6
   10 IF(ABS(Z(J)-X)-ABS(Z(I)-X))12,12,11
   11 J=I
C     TABLE SELECTION
   12 K=J
      JL=0
      JR=0
      DO 20 I=1,N
      ARG(I)=Z(K)
      IF(ICOL-1)14,14,13
   13 VAL(2*I-1)=F(K)
      KK=K+IROW
      VAL(2*I)=F(KK)
      GO TO 15
   14 VAL(I)=F(K)
   15 JJR=J+JR
      IF(JJR-IROW)16,18,18
   16 JJL=J-JL
      IF(JJL-1)19,19,17
   17 IF(ABS(Z(JJR+1)-X)-ABS(Z(JJL-1)-X))19,19,18
   18 JL=JL+1
      K=J-JL
      GO TO 20
   19 JR=JR+1
      K=J+JR
   20 CONTINUE
      RETURN
```

```
C       CASE IFOW=1
    21  ARG(1)=Z(1)
        VAL(1)=F(1)
        IF(ICOL-2)23,22,23
    22  VAL(2)=F(2)
    23  RETURN
        END

        SUBROUTINE ONE(X,ARG,VAL,Y,NDIM,EPS,IER)

        DIMENSION ARG(1),VAL(1)
        IER=2
        H2=X-ARG(1)
        IF(NDIM-1)2,1,3
     1  Y=VAL(1)+VAL(2)*H2
     2  RETURN
C
C       VECTOR ARG HAS MORE THAN 1 ELEMENT.
C       THE FIRST STEP PREPARES VECTOR VAL SUCH THAT AITKEN
        SCHEME CAN BE USED.
     3  I=1
        DO 5 J=2,NDIM
        H1=H2
        H2=X-ARG(J)
        Y=VAL(I)
        VAL(I)=Y+VAL(I+1)*H1
        H=H1-H2
        IF(H)4,13,4
     4  VAL(I+1)=Y+(VAL(I+2)-Y)*H1/H
     5  I=I+2
        VAL(I)=VAL(I)+VAL(I+1)*H2
C       END OF FIRST STEP

C       PREPARE AITKEN SCHEME
        DELT2=0.
        IEND=I-1

C       START AITKEN-LOOP
        DO 9 I=1,IEND
        DELT1=DELT2
        Y=VAL(1)
        M=(I+3)/2
        H1=ARG(M)
        DO 6 J=1,I
        K=I+1-J
        L=(K+1)/2
        H=ARG(L)-H1
        IF(H)6,14,6
     6  VAL(K)=(VAL(K)*(X-H1)-VAL(K+1)*(X-ARG(L)))/H
        DELT2=ABS(Y-VAL(1))
        IF(DELT2-EPS)11,11,7
     7  IF(I-5)9,8,3
     8  IF(DELT2-DELT1)9,12,12
     9  CONTINUE
C       END OF AITKEN-LOOP

    10  Y=VAL(1)
        RETURN

        THERE IS SUFFICIENT ACCURACY WITHIN 2*NDIM-2 ITERATION
        STEPS
    11  IER=0
        GOTO 10

C       TEST VALUE DELT2 STARTS OSCILLATING
    12  IER=1
        RETURN

C       THERE ARE TWO IDENTICAL ARGUMENT VALUES IN VECTOR ARG
    13  Y=VAL(1)
    14  IER=3
```

```
      RETURN
      END

      SUBROUTINE TWO(Y,PY,YT,PYT)

      DIMENSION YT(25),PYT(25)
      IF(Y.LT.YT(1)) GO TO 301
      J=2
  110 IF(Y-YT(J)) 200,201,202
  202 J=J+1
      IF(J.LE.25) GO TO 110
      PY=1.0
      GO TO 300
  201 PY=PYT(J)
      GO TO 300
  200 PY=PYT(J-1)+(PYT(J)-PYT(J-1))/(YT(J)-YT(J-1))*(Y-YT(J-
      GO TO 300
  301 PY=0.0
  300 RETURN
      END
```

PHI	CUMULATIVE DISTRIBUTION
0.57087100E 00	0.62946975E-01
0.57287097E 00	0.68146110E-01
0.57487094E 00	0.74001014E-01
0.57687092E 00	0.79798685E-01
0.57887089E 00	0.85878432E-01
0.58087087E 00	0.92248380E-01
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13. ABSTRACT

Techniques for extrapolating statistical data on the parameters of individual components to the statistical performance index for an overall system are considered. Two cases are evaluated. (1) The deterministic case in which the system's performance index is known functionally in terms of the system parameters. (2) The non-deterministic case in which only limited data on the performance index and its sensitivity with respect to system parameters is known. Computer programs are developed in both cases for combining given probability density distributions for the parameters into an overall probability density distribution for the system performance index. Theory and programs are developed and verified with a specific numerical example.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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Deterministic						
Jacobian						
Non-deterministic						
Probability Density						
Sensitivity Coefficient						